Lake Como School of Advanced Studies

Mathematical Analysis and Applications

June 9–13, 2025

Program of the courses

Poincaré-type and fundamental gap inequalities

Ilaria Fragalà

Abstract: I will review some celebrated inequalities for the Poincaré constant and for the fundamental gap of convex domains in the Euclidean space, and I will discuss their refined versions recently obtained in a joint research project with Vincenzo Amato (University of Naples) and Dorin Bucur (University of Savoie-Mont Blanc).

References:

- L. E. Payne and H. F. Weinberger, *An optimal Poincaré inequality for convex domains,* Arch. Rational Mech. Anal. 5 (1960).
- B. Andrews and J. Clutterbuck, *Proof of the fundamental gap conjecture,* J. Amer. Math. Soc. 24 (2011), no. 3, 899–916.
- V. Amato, D. Bucur, I. Fragalà, *The geometric size of the fundamental gap,* arXiv:2407.01341.
- V. Amato, D. Bucur, I. Fragalà, *A sharp quantitative nonlinear Poincaré inequality on convex domains,* arXiv:2407.20373.

Sobolev Spaces: Classical Foundations and Modern Perspectives

Giovanni Leoni

Abstract: In these lectures, I will revisit some classical results on Sobolev spaces. I will then discuss recent developments, including modern techniques and perspectives, and conclude with open problems.

References:

- R.A. Adams and J. J. F. Fournier, *Sobolev Spaces,* Academic Press, 2003.
- G. Leoni, *A First Course in Sobolev Spaces,* Second edition. AMS, 2017.
- G. Leoni, *A First Course in Fractional Sobolev Spaces,* AMS, 2023.
- G. Leoni and D. Spector, *On the Trace of* $\dot{W}_a^{m+1,1}(\mathbb{R}_+^{n+1})$, (2024), preprint.
- G. Leoni and I. Tice, *Traces for homogeneous Sobolev spaces in infinite strip-like domains.* J. Funct. Anal. 277 (2019), no. 7, 2288-2380.

Analysis of and evolutionary Gamma convergence for gradient systems

Alexander Mielke

Abstract: The first part consists of an introduction to gradient systems and their evolution, often also called gradient flows. We start from the basic existence theory via the minimizing movement scheme and discuss convergence results in Hilbert, Banach, and metric spaces. This leads to different notions of solutions such as EDB solutions, curves of maximal slope, and EVI solutions. We will explain a few of the main ideas in [AGS05, San17, Mie23] via simple examples.
The second part discusses the convergence of solutions when the functional and the geometric structure depend on a small parameter, which leads to questions of evolutionary Gamma convergence, as developed in [SaS04, Mie16] and refined by the notion of EDP convergence in [DFM19, MMP21].

References:

[AGS05] L. Ambrosio, N. Gigli, and G. Savaré, *Gradient flows in metric spaces and in the space of probability measures*, Lectures in Mathematics ETH Zürich, Birkhäuser Verlag, Basel, 2005.

[DFM19] P. Dondl, T. Frenzel, and A. Mielke: *A gradient system with a wiggly energy and relaxed EDP-convergence*. ESAIM Control Optim. Calc. Var. **25** (2019) 68/1–45.

[Mie16] A. Mielke, *On evolutionary Γ-convergence for gradient systems (Ch. 3)*. In "Macroscopic and Large Scale Phenomena: Coarse Graining, Mean Field Limits and Ergodicity (A. Muntean, J. Rademacher, and A. Zagaris, eds.), Lecture Notes in Applied Math. Mechanics Vol. 3, Springer, 2016", pp. 187–249.

[Mie23] ———, *An introduction to the analysis of gradient systems*, WIAS Preprint 3022, arXiv:2306.05026, 2023, (Script of a lecture course 2022/23, 100 pp.).

[MMP21] A. Mielke, A. Montefusco, and M. A. Peletier: *Exploring families of energy-dissipation landscapes via tilting — three types of EDP convergence*. Contin. Mech. Thermodyn. **33** (2021) 611–637.

[San17] F. Santambrogio: *{Euclidean, metric, Wasserstein} gradient flows: an overview*. Bull. Math. Sci. **7**:1 (2017) 87–154.

[SaS04] E. Sandier and S. Serfaty: *Gamma-convergence of gradient flows with applications to Ginzburg-Landau*. Comm. Pure Appl. Math. **LVII** (2004) 1627–1672.